Emergence of power laws in hierarchical dynamics on multi-level graphs

Tommaso Rondini*, Gregorio Berselli, Mirko Degli Esposti and Armando Bazzani

Department of Physics and Astronomy, DIFA, via Irnerio 46, 40126 Bologna, Italy

INFN, Bologna section, viale Berti Pichat 6/2, 40127 Bologna, Italy

tommaso.rondini2@unibo.it, gregorio.berselli2@unibo.it

City Science Lab

Bologna, Italy

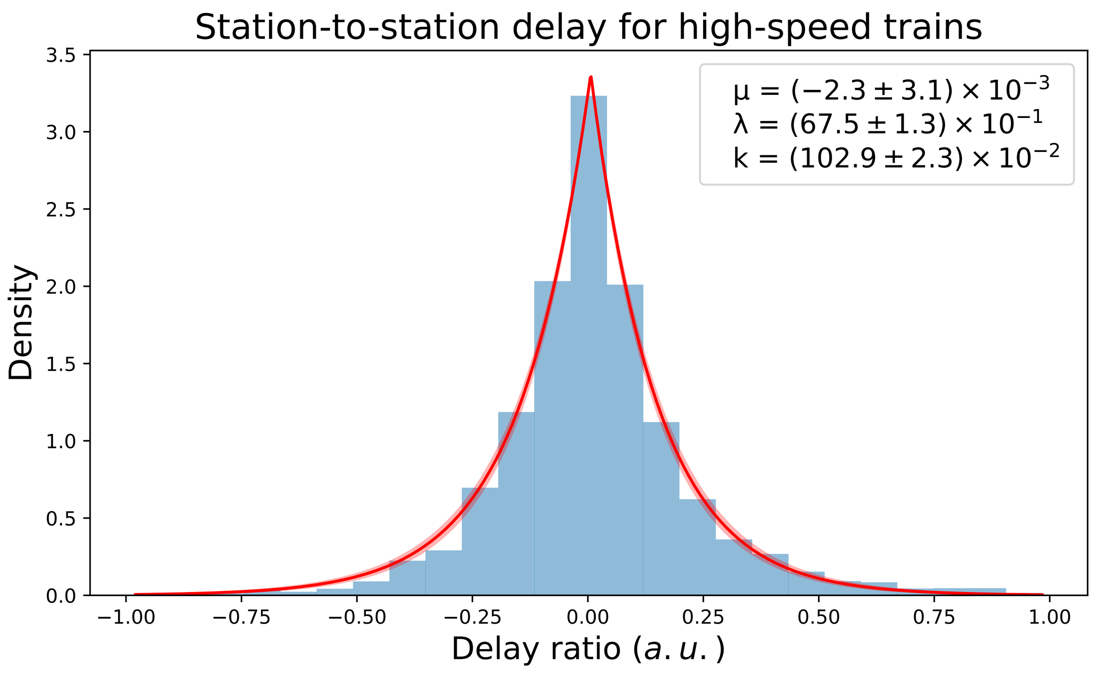
1. Abstract

Power-law distributions are widely recognized in complex systems' physics as indicative of underlying complexity of interaction networks and critical macroscopic behavior. In this work, we examine how power-law behavior arises in delay distributions within a multi-level hierarchical network of moving agents in presence of simple **priority rules**. Using the Italian railway systems as case study, we introduce Laplacian fluctuations into the dynamics and find that **local trains** experience significantly more frequent and larger delays than **high-speed ones** which can be interpolated by a power-law. We propose a queue-based dynamical model which accurately reproduces the observed power-law exponent characterizing the Italian local train delays.

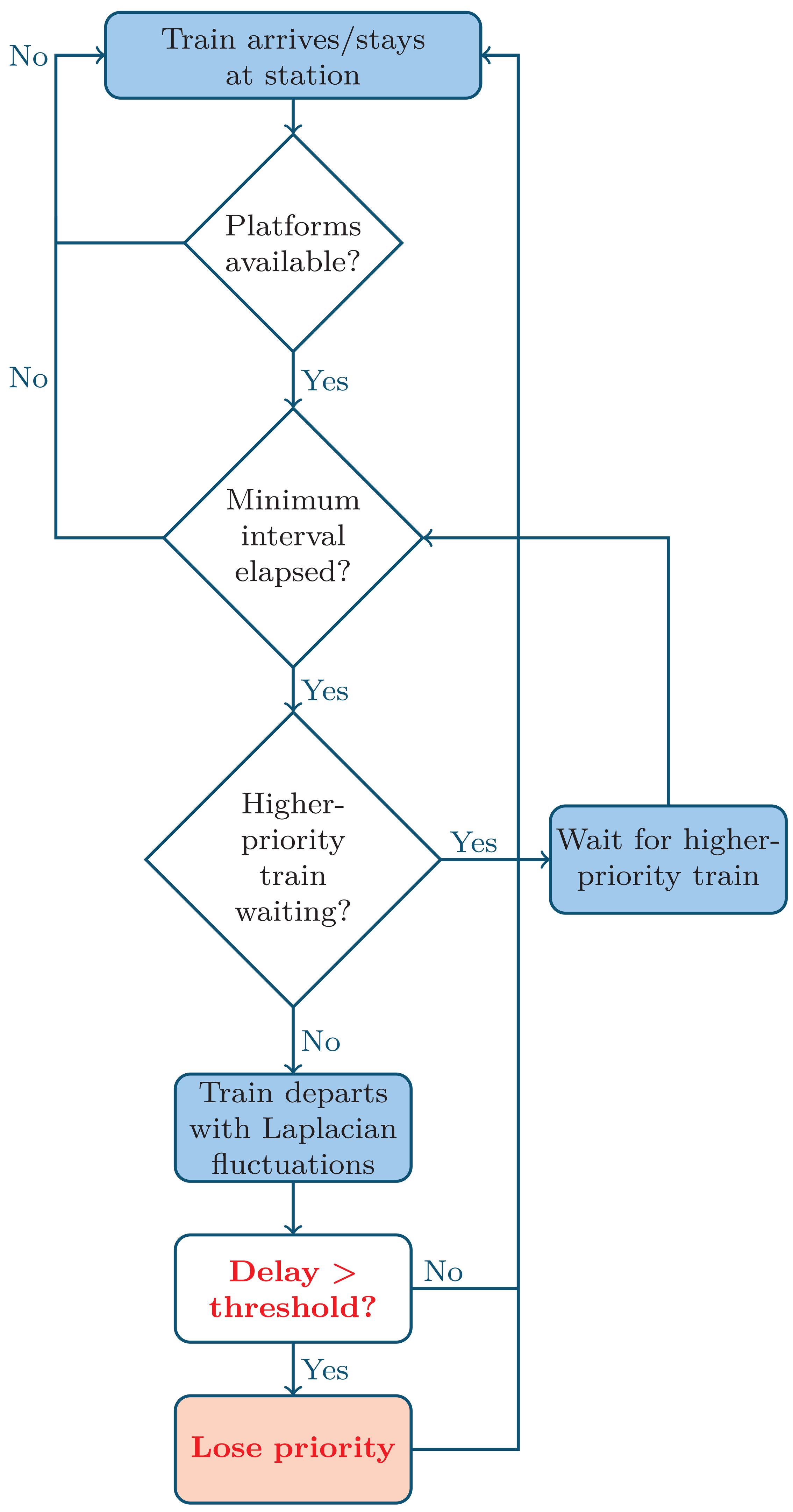
2. Real Data

We collected scheduled trips data to build the railway network, introducing many connections between stations.

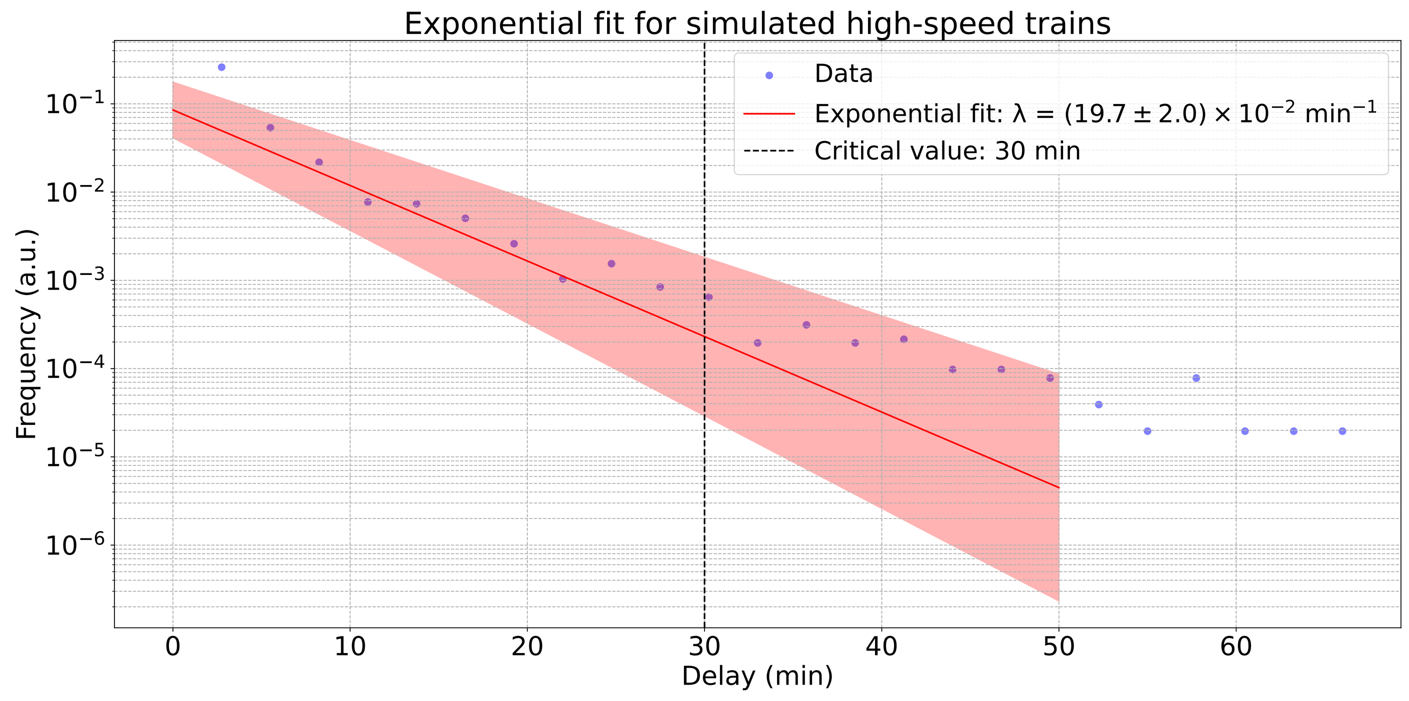
An **Asymmetric Laplacian** distribution emerges as sum of exponential distributions.

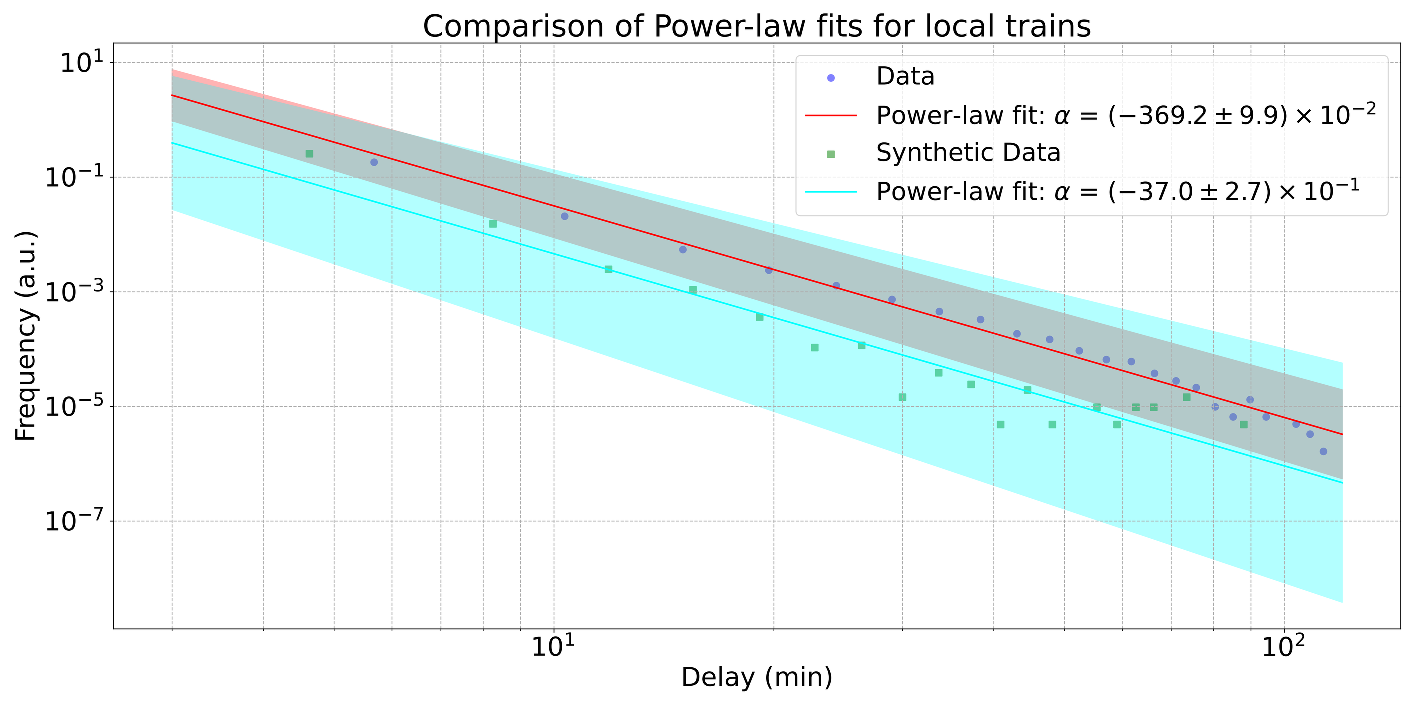


3. Queuing Model



4. Results





We simulated the same number of days as in the empirical dataset to obtain comparable statistics.

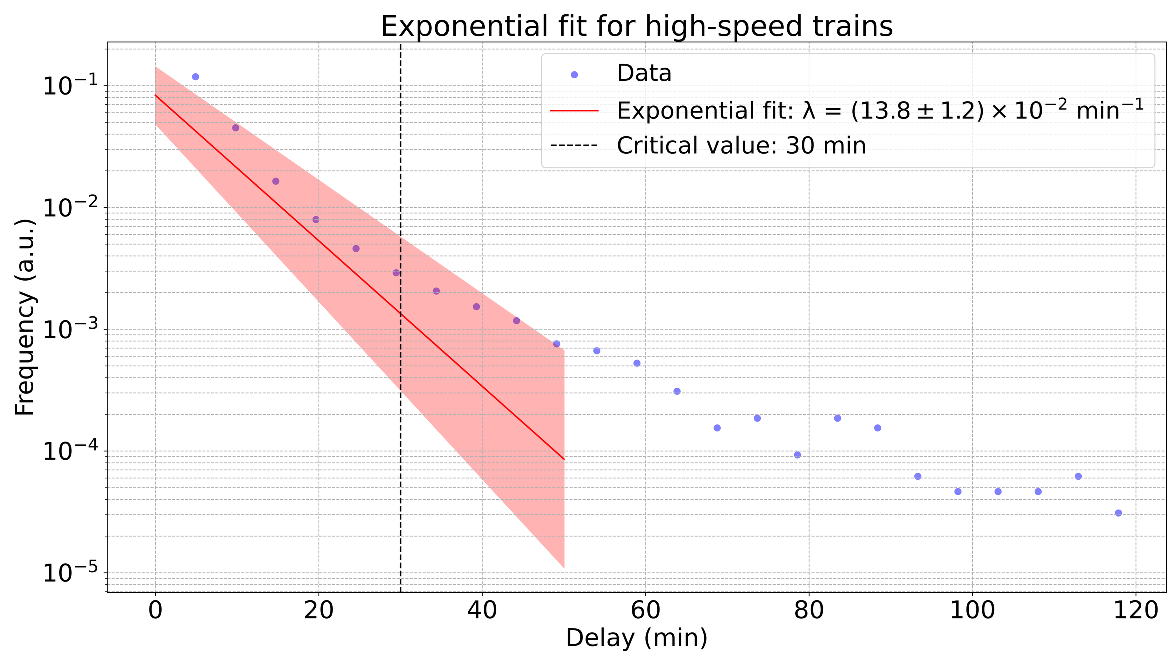
High-speed trains follow an **exponential distribution up to a critical delay threshold**, after which operational policies impose a loss of priority. The fit is computed up to that threshold and then plotted over a wider range to highlight the tail deviation.

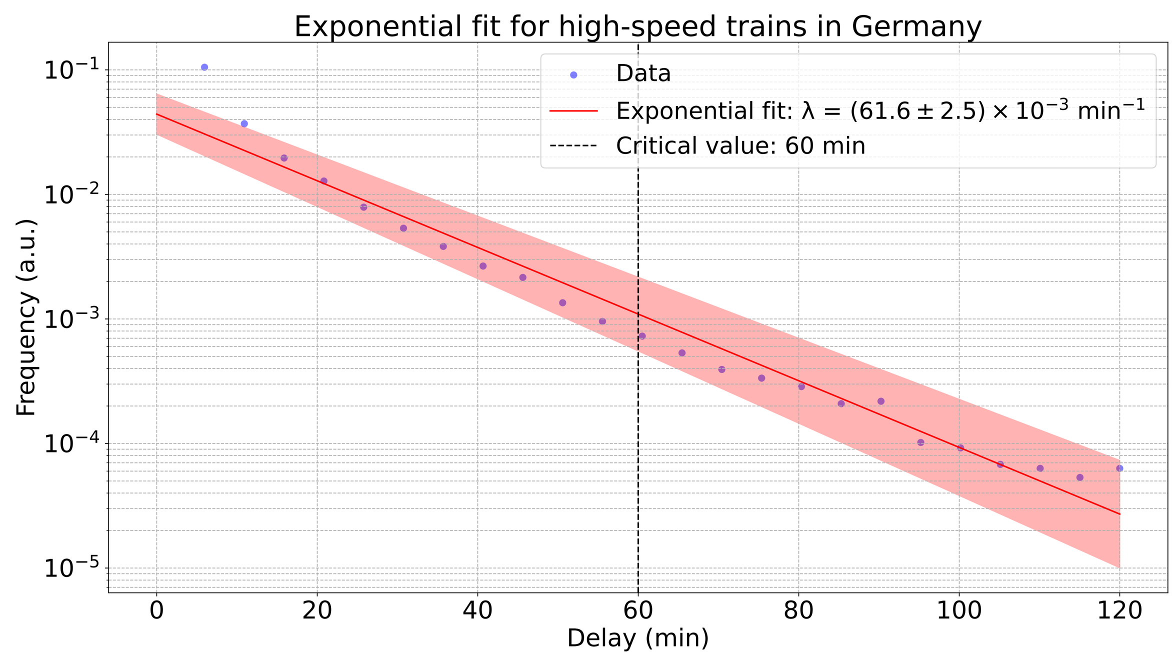
Local trains exhibit a power-law delay distribution with an exponent **compatible with the empirical data**. The fit is estimated over the entire range of observed delays.

Both graphs exclude extreme data points (attributable to factors such as accidents or strikes) to focus on routine operational behavior.

5. Comparison with other countries

We compared empirical delay distributions from the **German railway** network with those from Italy. Both systems exhibit **analogous behavior**: high-speed trains display an exponential delay distribution, whereas local trains follow a power-law distribution.





The key distinction lies in operational policy: in Germany, priority is **not revoked** even for large delays (as shown in the right-hand graph), so high-speed train delays remain exponential across the full data range, with a lower decay rate. Conversely, **local trains** still obey a power-law distribution, but with a **shallower exponent** (Italian: -3.69, German: -2.74) resulting in a heavier tail.

6. Conclusions

In this work, we have shown that **simple hierarchical rules**, implemented via priority-based queue dynamics on a multi-level network, are sufficient to **explain the heavy-tailed delay** distributions observed in real-world railway systems.

These findings confirm that minimal rule sets, rather than elaborate interaction kernels or finely tuned network topologies, can generate macroscopic phenomena such as power-law delays. A direct comparison between the Italian and German datasets further highlights how **operational policies imprint themselves on delay statistics**.

7. Bibliography

Sornette D., *Dragon-Kings, Black Swans and the Prediction of Crises, International Journal of Modern Physics C* **20(1)** (2009) 1-17

Newman M., *Power laws, Pareto distributions and Zipf's law, Contemporary Physics* **46(5)** (2005) 323-351

Monechi B., Gravino P. et al., *Complex delay dynamics on railway networks from universal laws to realistic modelling, EPJ Data Science* **7** (2018) 35

Grether D., Neumann A., Nagel K., *Simulation of Urban Traffic Control: A Queue Model Approach, Procedia Computer Science* **10** (2012) 808-814